

(Analytic Formula for the European Normal Black Scholes Formula)

by

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In this short summary paper, a brief summary of Black Scholes type formulae for Normal model will be given. Usually the underlying security is assumed to follow a lognormal process (or Geometric Brownian Motion). However, there are some traders who believe that the normal process describes the real market more closely than that of lognormal counterpart. The derivation of the formula will be proved mathematically by the famous no-arbitrage argument (Hull[1]). Then, the Greeks (delta, gamma, theta, and vega) will be computed by differentiation.

The idea of the theory is that The fair value of any derivative security is computed as the expectation of the payoff under an equivalent martingale measure (Karatzas [2]). More specifically, the price is the expectation of a discounted payoff under the risk neutral measure. This measure (or density) is a solution to a parabolic partial differential equation driven by the underlying process (lognormal process). The detailed statements will be shown below. Section following the next will show arguments concerning Normal model that does not allow negative underlier price. Also brief experiments regarding how one can approximate lognormal Black Scholes with Normal version will be explored with some experiments.

1 Analytic Formula

Theorem 1 (Analytic Formula for a Normal Black Scholes Model)

Let us assume that the current future price, strike price, risk free interest rate, volatility, and time to maturity as denoted as F , K , r , σ , and $T - t$ respectively. Alet us also assume that the current future price follows the following Normal process:

$$dF = \mu dt + \sigma dW_t \quad (1)$$

where μ is a constant drift.

Then, the fair values of call C and put P are expressed as :

$$C = e^{-r(T-t)}[(F - K)N(d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}] \quad (2)$$

and

$$P = e^{-r(T-t)}[(K - F)N(-d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}] \quad (3)$$

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where $d_1 = \frac{F-K}{\sigma\sqrt{T-t}}$

Moreover, the Greeks (delta, gamma, vega, theta) are computed by simple differentiation of the above formulas to give: (Note: Detail calculations will be shown below for more general audience)

$$\begin{aligned}
Delta_{(Call)} &= \frac{\partial C}{\partial F} \\
&= e^{-r(T-t)} \left[(F-K) \frac{1}{\sqrt{2\pi}} e^{-d_1^2/2} \frac{1}{\sigma\sqrt{T-t}} \right] \\
&\quad + e^{-r(T-t)} N(d_1) + e^{-r(T-t)} \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}} e^{-d_1^2/2} \left[-\frac{(F-K)}{\sigma\sqrt{T-t}} \right] \left[\frac{1}{\sigma\sqrt{T-t}} \right] \\
&= e^{-r(T-t)} N(d_1)
\end{aligned} \tag{4}$$

$$\begin{aligned}
Delta_{(Put)} &= \frac{\partial P}{\partial F} \\
&= e^{-r(T-t)} \left[(K-F) \frac{1}{\sqrt{2\pi}} e^{-d_1^2/2} \frac{-1}{\sigma\sqrt{T-t}} \right] \\
&\quad - e^{-r(T-t)} N(-d_1) + e^{-r(T-t)} \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}} e^{-d_1^2/2} \left[-\frac{(F-K)}{\sigma\sqrt{T-t}} \right] \left[\frac{1}{\sigma\sqrt{T-t}} \right] \\
&= -e^{-r(T-t)} N(-d_1)
\end{aligned} \tag{5}$$

$$\begin{aligned}
Gamma_{(Call)} = Gamma_{(Put)} &= \frac{\partial^2 C}{\partial F^2} (*) \\
&= e^{-r(T-t)} \frac{1}{\sqrt{2\pi}} e^{-d_1^2/2} \frac{1}{\sigma\sqrt{T-t}} \\
&= \frac{e^{-r(T-t)}}{\sigma\sqrt{T-t}} \frac{1}{\sqrt{2\pi}} e^{-d_1^2/2}
\end{aligned} \tag{6}$$

(*Note: This is evident from the Put-Call parity in that by differentiating the Put-Call parity formula twice with respect to the underlier establishes the equality of Put and Call for all option models ..)

$$\begin{aligned}
Vega_{(Call)} = Vega_{(Put)} &= \frac{\partial C}{\partial \sigma} \\
&= e^{-r(T-t)} (F-K) \frac{1}{\sqrt{2\pi}} e^{-d_1^2/2} \left[-\frac{(F-K)}{\sigma^2\sqrt{T-t}} \right] \\
&\quad + e^{-r(T-t)} \frac{\sqrt{T-t}}{\sqrt{2\pi}} e^{-d_1^2/2} \\
&\quad + e^{-r(T-t)} \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}} e^{-d_1^2/2} \left[-\frac{(F-K)}{\sigma\sqrt{T-t}} \right] \left[-\frac{(F-K)}{\sigma^2\sqrt{T-t}} \right] \\
&= e^{-r(T-t)} \frac{\sqrt{T-t}}{\sqrt{2\pi}} e^{-d_1^2/2}
\end{aligned} \tag{7}$$

(*Note: This is evident from the Put-Call parity in that by differentiating the Put-Call parity formula with respect to σ establishes the equality of Put and Call for all option models ..)

$$\begin{aligned}
Theta_{(Call)} &= \frac{\partial C}{\partial t} \\
&= re^{-r(T-t)}[(F-K)N(d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}] \\
&\quad + e^{-r(T-t)}[(F-K)\frac{1}{\sqrt{2\pi}}e^{-d_1^2/2}(\frac{1}{2}\frac{F-K}{\sigma(T-t)^{3/2}})] \\
&\quad - e^{-r(T-t)}[\frac{\sigma}{2\sqrt{2\pi}(T-t)}e^{-d_1^2/2}] \\
&\quad + e^{-r(T-t)}[\frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}(-\frac{(F-K)}{\sigma\sqrt{T-t}})(\frac{(F-K)}{2\sigma(T-t)^{3/2}})] \\
&= re^{-r(T-t)}[(F-K)N(d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}] \\
&\quad - e^{-r(T-t)}[\frac{\sigma}{2\sqrt{2\pi}(T-t)}e^{-d_1^2/2}] \tag{8}
\end{aligned}$$

$$\begin{aligned}
Theta_{(Put)} &= \frac{\partial P}{\partial t} \\
&= re^{-r(T-t)}[(K-F)N(-d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}] \\
&\quad + e^{-r(T-t)}[(K-F)\frac{1}{\sqrt{2\pi}}e^{-d_1^2/2}(\frac{1}{2}\frac{F-K}{\sigma(T-t)^{3/2}})] \\
&\quad - e^{-r(T-t)}[\frac{\sigma}{2\sqrt{2\pi}(T-t)}e^{-d_1^2/2}] \\
&\quad + e^{-r(T-t)}[\frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}(-\frac{(F-K)}{\sigma\sqrt{T-t}})(\frac{(F-K)}{2\sigma(T-t)^{3/2}})] \\
&= re^{-r(T-t)}[(K-F)N(-d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}] \\
&\quad - e^{-r(T-t)}[\frac{\sigma}{2\sqrt{2\pi}(T-t)}e^{-d_1^2/2}] \tag{9}
\end{aligned}$$

(Proof)

We shall parallel the argument given in Hull [1]. The proof will be heuristic. Let us study the behavior of the delta hedged portfolio which consists of long delta shares of future contract and short one derivative in question. Say, call it Π . Let us also denote the value of derivative by g . Then, the value of the delta hedged portfolio is given by:

$$\Pi = g - \frac{\partial g}{\partial F}F \tag{10}$$

So applying Ito's lemma using the SDE given in (1) into the changes of the above portfolio value, one can get:

$$\begin{aligned}
\Delta \Pi &= \Delta g - \frac{\partial g}{\partial F} \Delta F \\
&= \left(\frac{\partial g}{\partial t} + \frac{\partial g}{\partial F} \mu + \frac{1}{2} \frac{\partial^2 g}{\partial F^2} \sigma^2 \right) \Delta t + \frac{\partial g}{\partial F} \sigma \Delta W_t \\
&\quad - \frac{\partial g}{\partial F} (\mu \Delta t + \sigma \Delta W_t) \\
&= \left(\frac{\partial g}{\partial t} + \frac{1}{2} \frac{\partial^2 g}{\partial F^2} \sigma^2 \right) \Delta t
\end{aligned} \tag{11}$$

We want the above quantity to be a martingale under the discounted expectation with risk free rate. This is essentially the same as stating that the above quantity equals the gain from the risk free interest rate for the portfolio value. So, we have:

$$\Delta \Pi = r \Pi \Delta t \tag{12}$$

Since it cost nothing to enter into a futures contract, one has:

$$\Pi = g \tag{13}$$

Thus we have:

$$\left(\frac{\partial g}{\partial t} + \frac{1}{2} \frac{\partial^2 g}{\partial F^2} \sigma^2 \right) \Delta t = r g \Delta t \tag{14}$$

Therefore, we obtains the following parabolic PDE:

$$\frac{\partial g}{\partial t} + \frac{1}{2} \frac{\partial^2 g}{\partial F^2} \sigma^2 = r g \tag{15}$$

So, if the terminal payoff is given by $g(T, F) = f(F)$, then by the application of Feynman-Kac (see Karatzas and Shrieve[2]), one obtains the following solution:

$$\begin{aligned}
g(t, x) &= E_x [e^{-r(T-t)} f(y)] \\
&= \frac{e^{-r(T-t)}}{\sigma \sqrt{T-t} \sqrt{2\pi}} \int_{-\infty}^{\infty} f(y) e^{-\frac{(x-y)^2}{2\sigma^2(T-t)}} dy
\end{aligned} \tag{16}$$

where

$$f(y) = \begin{cases} (y - K)^+ & \text{for Call} \\ (K - y)^+ & \text{for Put.} \end{cases}$$

Therefore, the for the formula for the above can be simplified by simply expanding the expression inside the integral. The detail will be shown for more general audience.

For the call, we have:

$$\begin{aligned}
Call &= g(t, F) \\
&= E_F[e^{-r(T-t)} f(y)] \\
&= \frac{e^{-r(T-t)}}{\sigma\sqrt{T-t}\sqrt{2\pi}} \int_{-\infty}^{\infty} (y-K)^+ e^{-\frac{(F-y)^2}{2\sigma^2(T-t)}} dy \\
&= \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (F + \sigma\sqrt{T-t}x - K)^+ e^{-\frac{x^2}{2}} dx \\
&= \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{\frac{(K-F)}{\sigma\sqrt{T-t}}}^{\infty} (F + \sigma\sqrt{T-t}x - K) e^{-\frac{x^2}{2}} dx \\
&= \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{\frac{(K-F)}{\sigma\sqrt{T-t}}}^{\infty} (F-K) e^{-\frac{x^2}{2}} dx \\
&+ \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{\frac{(K-F)}{\sigma\sqrt{T-t}}}^{\infty} (\sigma\sqrt{T-t}x) e^{-\frac{x^2}{2}} dx \\
&= e^{-r(T-t)} [(F-K)N(d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}} e^{-d_1^2/2}] \tag{17}
\end{aligned}$$

where $d_1 = \frac{F-K}{\sigma\sqrt{T-t}}$

Similarly for the put, we have:

$$\begin{aligned}
Put &= g(t, F) \\
&= E_F[e^{-r(T-t)} f(y)] \\
&= \frac{e^{-r(T-t)}}{\sigma\sqrt{T-t}\sqrt{2\pi}} \int_{-\infty}^{\infty} (K-y)^+ e^{-\frac{(F-y)^2}{2\sigma^2(T-t)}} dy \\
&= \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (K-F - \sigma\sqrt{T-t}x)^+ e^{-\frac{x^2}{2}} dx \\
&= \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{\frac{(K-F)}{\sigma\sqrt{T-t}}}^{\infty} (K-F - \sigma\sqrt{T-t}x) e^{-\frac{x^2}{2}} dx \\
&= \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{-\infty}^{\frac{(K-F)}{\sigma\sqrt{T-t}}} (K-F) e^{-\frac{x^2}{2}} dx \\
&+ \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{-\infty}^{\frac{(K-F)}{\sigma\sqrt{T-t}}} (-\sigma\sqrt{T-t}x) e^{-\frac{x^2}{2}} dx \\
&= e^{-r(T-t)} [(K-F)N(-d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}} e^{-d_1^2/2}] \tag{18}
\end{aligned}$$

where d_1 is defined similarly as in (17).

This is what we need to prove. The Greeks are obtained by simple differentiations which are worked out in the statement of the theorem.

QED.

2 Normal Process under the Bounded Negative Assumption

In the previous section, pricing scheme for the Normal Process is proved in more general setting. That is the underlying prices are allowed to become negative. In this section, the modified option formulas will be derived where underlier price can not fall below certain number, say $-R$.

Using the same notations as in the previous section, we have the following PDE for non negative underlier process.

$$\begin{aligned}\frac{\partial g}{\partial t} + \frac{1}{2} \frac{\partial^2 g}{\partial F^2} \sigma^2 &= rg \\ g(T, F) &= f(F) \\ F &\geq -R\end{aligned}\tag{19}$$

This is a problem constrained in the half plane in F axis. The solution of the above PDE can be solved by transforming the above PDE into a typical PDE problem for the entire F axis. This is done by extending the terminal function $f(F)$ into $f_{-R}(F)$ which is basically an odd extension of the original function $f(F)$.

Thus, the unique solution exists for this PDE (John [4]), and given by:

$$\begin{aligned}g(t, x) &= E_x[e^{-r(T-t)} f(y)] \\ &= \frac{e^{-r(T-t)}}{\sigma\sqrt{T-t}\sqrt{2\pi}} \int_{-\infty}^{\infty} f_{-R}(y) e^{-\frac{(x-y)^2}{2\sigma^2(T-t)}} dy \\ &= \frac{e^{-r(T-t)}}{\sigma\sqrt{T-t}\sqrt{2\pi}} \int_{-R}^{\infty} f(y) [e^{-\frac{(x-y+R)^2}{2\sigma^2(T-t)}} - e^{-\frac{(x+y-R)^2}{2\sigma^2(T-t)}}] dy\end{aligned}\tag{20}$$

where

$$f(y) = \begin{cases} (y - K)^+ & \text{for Call} \\ (K - y)^+ & \text{for Put.} \end{cases}$$

Therefore the formula for the Call option is given by expanding the messy expression in (20) as:

$$\begin{aligned}Call &= g(t, F) \\ &= E_F[e^{-r(T-t)} f(y)] \\ &= \frac{e^{-r(T-t)}}{\sigma\sqrt{T-t}\sqrt{2\pi}} \int_{-\infty}^{\infty} f_{-R}(y) e^{-\frac{(F-y)^2}{2\sigma^2(T-t)}} dy \\ &= \frac{e^{-r(T-t)}}{\sigma\sqrt{T-t}\sqrt{2\pi}} \int_{-R}^{\infty} f(y) [e^{-\frac{(F-y+R)^2}{2\sigma^2(T-t)}} - e^{-\frac{(F+y-R)^2}{2\sigma^2(T-t)}}] dy \\ &= \frac{e^{-r(T-t)}}{\sigma\sqrt{T-t}\sqrt{2\pi}} \int_{-R}^{\infty} (y - k)^+ [e^{-\frac{(F-y+R)^2}{2\sigma^2(T-t)}} - e^{-\frac{(F+y-R)^2}{2\sigma^2(T-t)}}] dy \\ &= \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{\frac{(K-F-R)}{\sigma\sqrt{T-t}}}^{\infty} (F + \sigma\sqrt{T-t}x + R - K)^+ e^{-\frac{x^2}{2}} dx \\ &\quad + \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{\frac{(K+F-R)}{\sigma\sqrt{T-t}}}^{\infty} (-F - \sigma\sqrt{T-t}z + R - K) e^{-\frac{z^2}{2}} dz \\ &= e^{-r(T-t)} [(R + F - K)N(d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}} e^{-d_1^2/2}]\end{aligned}$$

$$-e^{-r(T-t)}[(R - F - K)N(d_2) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_2^2/2}] \quad (21)$$

where $d_1 = \frac{R+F-K}{\sigma\sqrt{T-t}}$, and $d_2 = \frac{R-F-K}{\sigma\sqrt{T-t}}$

Similarly for put, we have:

$$\begin{aligned} Put &= g(t, F) \\ &= E_F[e^{-r(T-t)}f(y)] \\ &= \frac{e^{-r(T-t)}}{\sigma\sqrt{T-t}\sqrt{2\pi}} \int_{-\infty}^{\infty} f_{-R}(y)e^{-\frac{(F-y)^2}{2\sigma^2(T-t)}} dy \\ &= \frac{e^{-r(T-t)}}{\sigma\sqrt{T-t}\sqrt{2\pi}} \int_{-R}^{\infty} f(y)[e^{-\frac{(F-y+R)^2}{2\sigma^2(T-t)}} - e^{-\frac{(F+y-R)^2}{2\sigma^2(T-t)}}] dy \\ &= \frac{e^{-r(T-t)}}{\sigma\sqrt{T-t}\sqrt{2\pi}} \int_{-R}^{\infty} (k-y)^+ [e^{-\frac{(F-y+R)^2}{2\sigma^2(T-t)}} - e^{-\frac{(F+y-R)^2}{2\sigma^2(T-t)}}] dy \\ &= \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{-\infty}^{\frac{(K-F-R)}{\sigma\sqrt{T-t}}} (K-F-\sigma\sqrt{T-t}x-R)^+ e^{-\frac{x^2}{2}} dx \\ &\quad + \frac{e^{-r(T-t)}}{\sqrt{2\pi}} \int_{\frac{(K+F-R)}{\sigma\sqrt{T-t}}}^{\infty} (K+F-\sigma\sqrt{T-t}z-R)e^{-\frac{z^2}{2}} dz \\ &= e^{-r(T-t)}[(K-F-R)N(d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}] \\ &\quad - e^{-r(T-t)}[(K+F-R)N(d_2) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_2^2/2}] \end{aligned} \quad (22)$$

where $d_1 = \frac{R+F-K}{\sigma\sqrt{T-t}}$, and $d_2 = \frac{R-F-K}{\sigma\sqrt{T-t}}$

Let us compare the result with the results from the previous section. Letting $R = 0$ in the expression (21) gives:

$$\begin{aligned} Call &= e^{-r(T-t)}[(F-K)N(d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}] \\ &\quad - e^{-r(T-t)}[(-F-K)N(d_2) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_2^2/2}] \end{aligned} \quad (23)$$

where $d_1 = \frac{F-K}{\sigma\sqrt{T-t}}$, and $d_2 = \frac{-F-K}{\sigma\sqrt{T-t}}$

Since d_2 term is such a large negative quantity, the term $-e^{-r(T-t)}[(-F-K)N(d_2) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_2^2/2}]$ is a very small quantity.

Thus, the call price becomes:

$$Call \approx e^{-r(T-t)}[(F-K)N(d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}}e^{-d_1^2/2}] \quad (24)$$

which is the same result as the one given in the previous section.

Actually, the the second term is so small that one can replace the above relation with equality in most practical cases. The identical argument can be made for the put as well.

The above price have a very nice theoretical property when σ is extremely large. The expressions (2) and (3) blows up to infinity when we let $\sigma \rightarrow \infty$

Without the loss of generality, let us assume $R = 0$ as before. Then taking the limit of the expression (21) gives:

$$\begin{aligned}
\lim_{\sigma \rightarrow +\infty} Call &= \lim_{\sigma \rightarrow +\infty} e^{-r(T-t)} \left[(F - K)N(d_1) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}} e^{-d_1^2/2} \right] \\
&\quad \lim_{\sigma \rightarrow +\infty} -e^{-r(T-t)} \left[(-F - K)N(d_2) + \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}} e^{-d_2^2/2} \right] \\
&= e^{-r(T-t)}(F - K) + \lim_{\sigma \rightarrow +\infty} e^{-r(T-t)} \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}} [e^{-d_1^2/2} - e^{-d_2^2/2}] \\
&= e^{-r(T-t)}(F - K) + \lim_{\sigma \rightarrow +\infty} e^{-r(T-t)} \frac{\sigma\sqrt{T-t}}{\sqrt{2\pi}} \left[\sum_{n=1}^{\infty} \frac{\left(\left(\frac{-(F-K)^2}{2(T-t)} \right)^n - \left(\frac{-(F+K)^2}{2(T-t)} \right)^n \right)}{\sigma^{2n} n!} \right] \\
&= e^{-r(T-t)}(F - K)
\end{aligned} \tag{25}$$

Similarly for put, we have:

$$\lim_{\sigma \rightarrow +\infty} Put = e^{-r(T-t)}(K - F) \tag{26}$$

Thus, as the volatility σ blows up to infinity, the price is bounded above by constant for both call and put.

3 Approximation Experiments

In practice, option prices are given in the market and one ends up computing implied volatility for a particular option. Suppose one is given an implied volatility for the lognormal model but wants to use normal model to compute its greeks (delta, gamma, vega, and theta). This is usually dealt with approximating $\sigma_{(Normal)}$ by letting $\sigma_{(Normal)} = \hat{G}\sigma_{(Lognormal)}$, Where \hat{G} is a scaling constant. What kind approximations can one make? The popular choice is to set $\hat{G} = K$ (strike), or $\hat{G} = F$ (future price). Then use these into the Normal model. These approximations display reasonable closeness to the actual lognormal price. Brief experiments have shown that taking the average of these two $\hat{G} = (F + K)/2$ gives very approximation to the lognormal price. Since, this paper is intended to be a very short paper, brief explanation of the experiments with some graphs would be shown below.

The test is performed on European Eurodollar future option Call where strike is 98.25, days to expiration is 22 days, and risk free rate is set to 1.701 %. We took a ranges of market option prices from 0.05 to 0.13 and future prices from 97.8 to 98.3. For a given input price and a future price, Newton's method was used to compute Lognormal implied vol. Then this implied vol is scaled appropriately and inserted into Normal formulas.

Eurodollar Future Option Price Analysis (Using Spot as Scaling)

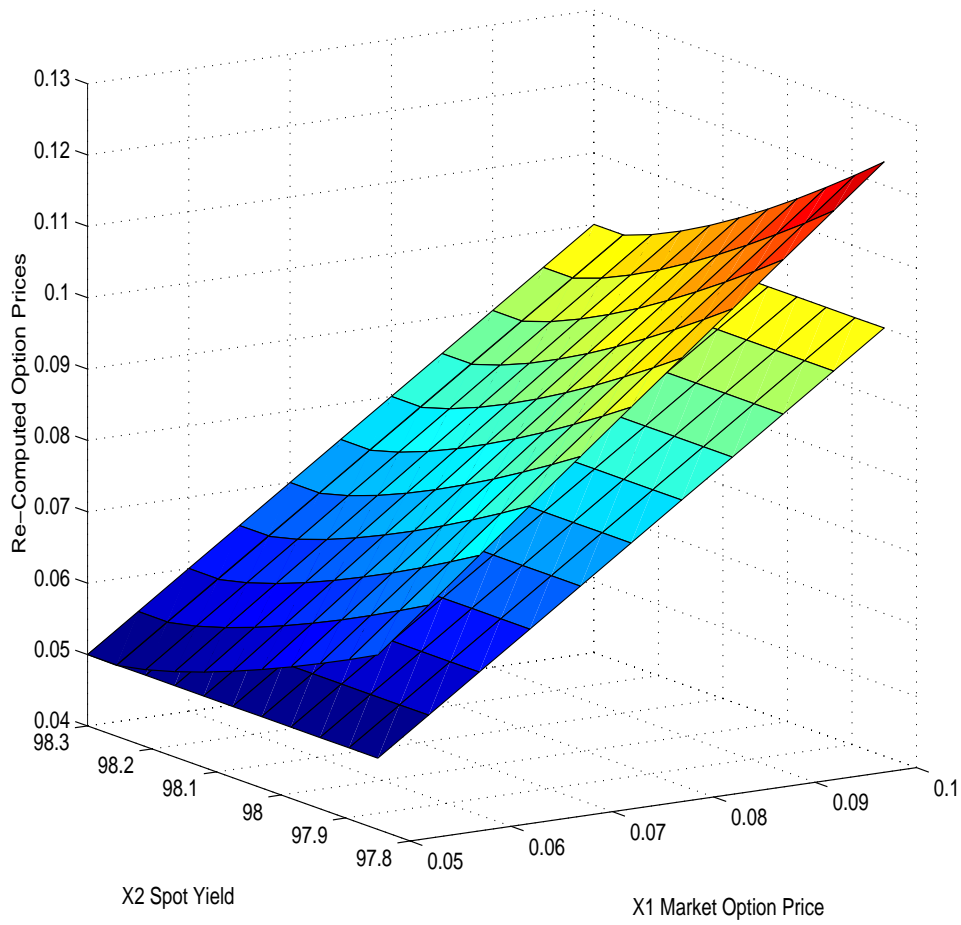


Figure 1: Eurodollar future Call. The top graph is the price surface using Normal model with 'spot' as the scaling factor. The bottom surface is the original Lognormal price surface. Note that Normal price is somewhat overstating the price.

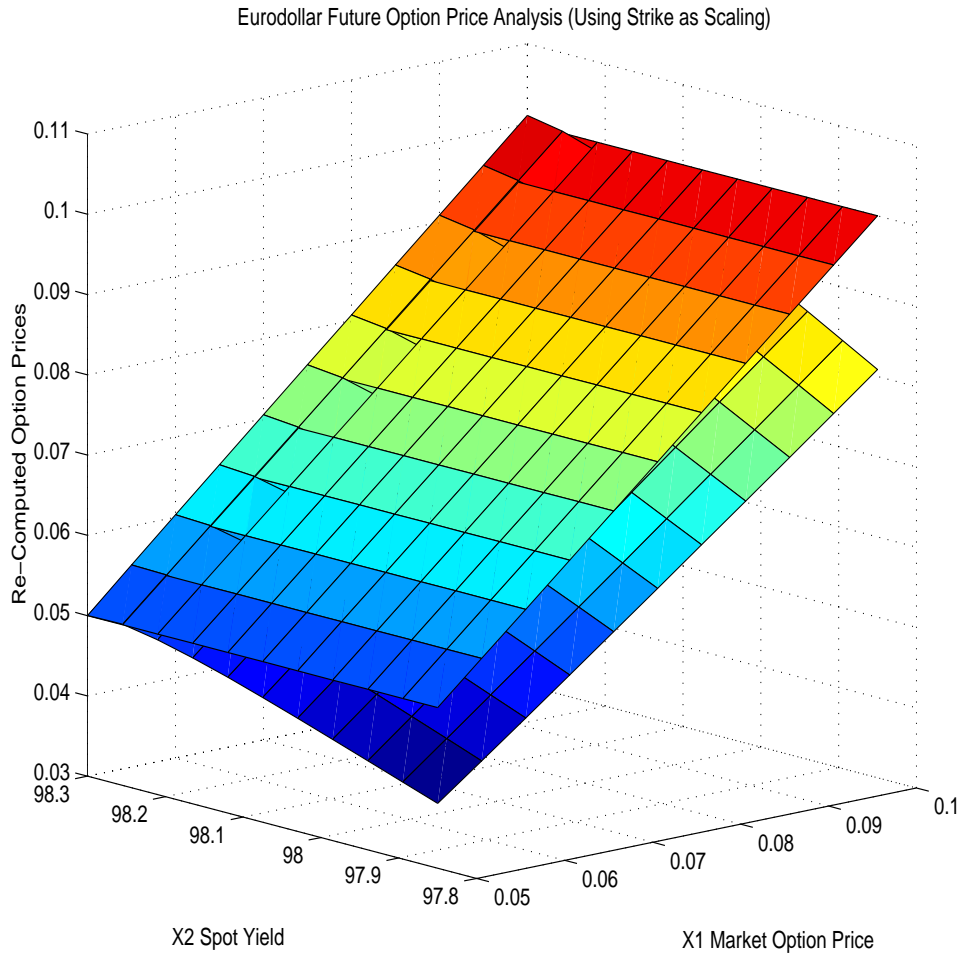


Figure 2: Eurodollar future Call. The top graph is the price surface using Normal model with 'strike' as the scaling factor. The bottom surface is the original Lognormal price surface. Note that Normal price is somewhat understating the price in this case.

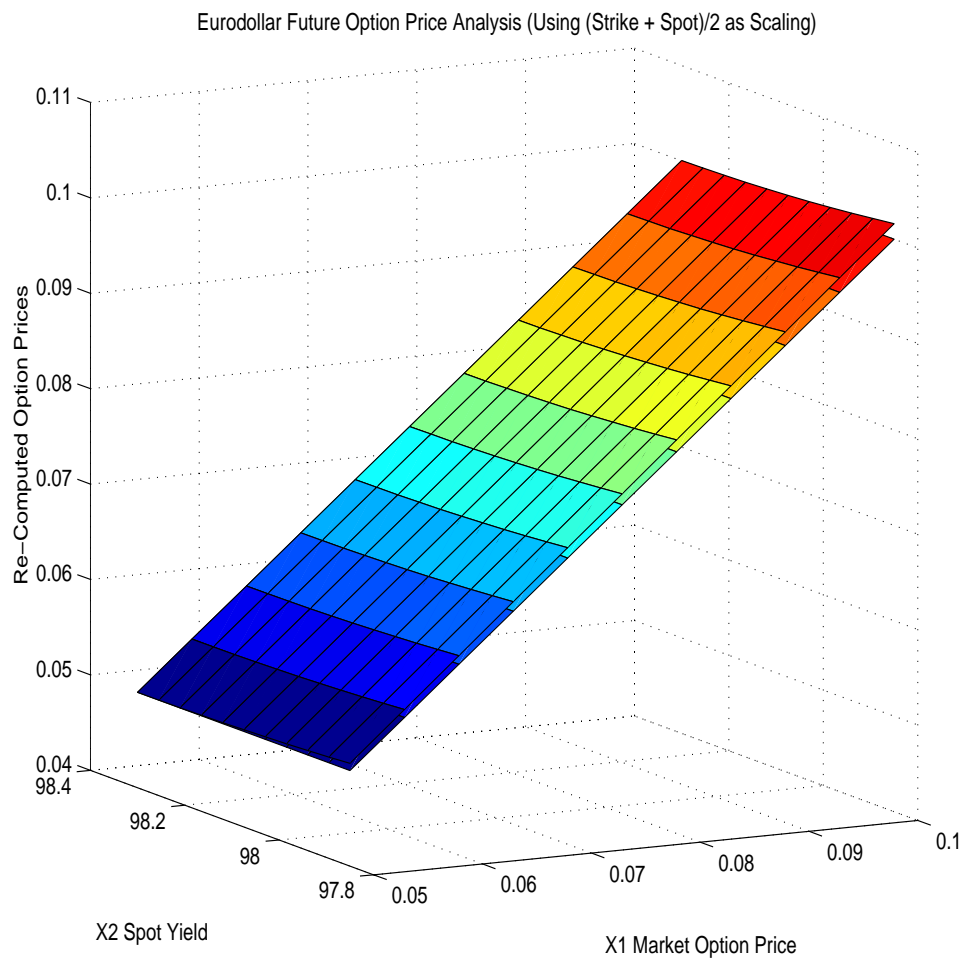


Figure 3: Eurodollar future Call. The top graph is the price surface using Normal model with ('strike' + 'spot')/2 as the scaling factor. The bottom surface is the original Lognormal price surface. Note that Normal price is much closer to the actual price in this case.

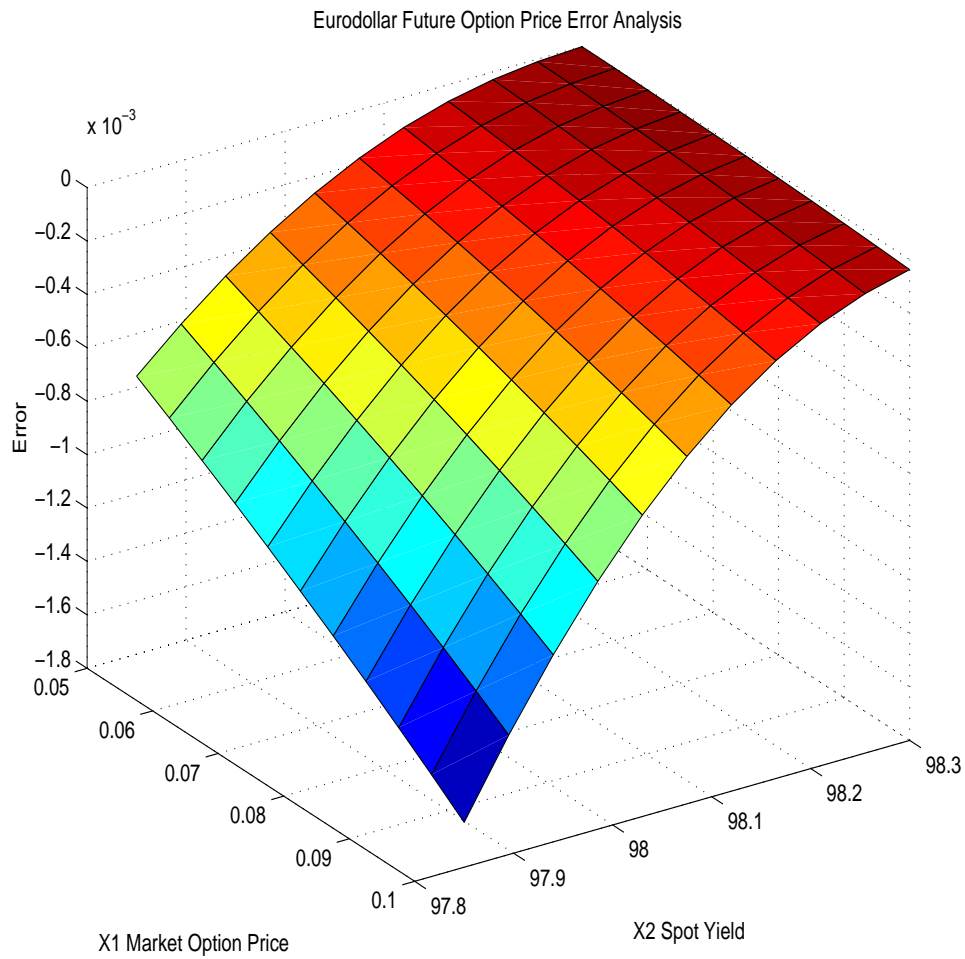


Figure 4: Eurodollar future Call. The error showing the difference between The Lognormal model and Normal model with $(\text{'strike'} + \text{'spot'})/2$ scaling. Note that the error increases as the option becomes far in the money.

References

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